NASA COOPERATIVE AGREEMENT NCCI-22

COMPUTATIONAL FLUID MECHANICS

Progress Report No. 27

For the Period June 15, 1992 to December 14, 1992

Prepared by

H. A. Hassan
Project Coordinator

Department of Mechanical and Aerospace Engineering
Post Office Box 7910
North Carolina State University
Raleigh, North Carolina 27695-7910

During this period a paper[1] was presented at the AIAA 10th Applied Aerodynamics Meeting. Later, the paper was submitted for publication in the AIAA Journal and was accepted. In addition, an abstract[2] was submitted to the 24th AIAA Fluid Dynamics Conference. Copy of the abstract is enclosed.

References

1. Young, T. W., Warren, E. S., Harris, J. E., and Hassan, H. A., "A New Approach for the Calculation of Translational Flows," AIAA Paper 92-2669, June 1992. To appear in the *AIAA Journal.*
2. Robinson, D. F., Harris, J. E., and Hassan, H. A., "Exploiting Similarity in Turbulent Shear Flows for Turbulence Modeling".

Exploiting Similarity in Turbulent Shear Flows for Turbulence Modeling

David F. Robinson*
North Carolina State University, Raleigh, North Carolina

Julius E. Harris†
NASA Langley Research Center, Hampton, Virginia

H. A. Hassan ‡
North Carolina State University, Raleigh, North Carolina

Abstract of Paper Proposed for the
24th AIAA Fluid Dynamics Conference
July 6-9, 1993
Orlando, Florida

Introduction

It is well known that current $k-\epsilon$ models cannot predict the flow over a flat plate and its wake[1-2]. In an effort to address this issue and other issues associated with turbulence closure, a new approach for turbulence modeling is proposed which exploits similarities in the flow field. Thus, if we consider the flow over a flat plate and its wake, then in addition to taking advantage of the log-law region, we can exploit the fact that the flow becomes self-similar in the far

*Research Assistant, Mechanical and Aerospace Engineering, Student Member AIAA.
†Senior research Scientist, Fluid Flow Physics Research., Fluid Dynamics Division, Associate Fellow AIAA.
‡Professor, Mechanical and Aerospace Engineering, Associate Fellow AIAA.

wake. This latter behavior makes it possible to cast the governing equations as a set of total differential equations. Solutions of this set and comparison with measured shear stress and velocity profiles yields the desired set of model constants. Such a set is, in general, different from other sets of model constants. The rational for such an approach is that if we can correctly model the flow over a flat plate and its far wake, then we can have a better chance of predicting the behavior in between. It is to be noted that the approach does not appeal, in any way, to the decay of homogeneous turbulence. This is because the asymptotic behavior of the flow under consideration is not representative of the decay of homogeneous turbulence.

Approach

The approach will be illustrated by a $k-\omega$ model. A Similar approach can be used for a $k-\epsilon$ model. For a plane wake, the defect velocity can be represented as[3]

$$U_e - u = U_s f\left(\frac{y}{l}\right) = U_s f(\eta) \tag{1}$$

where U_e is the free stream velocity, u is the velocity, and l is a length scale representative of the width of the wake. For the wake

$$U_s \sim x^{-\frac{1}{2}} \quad , \quad l \sim x^{\frac{1}{2}} \tag{2}$$

The k and ω equations can be written as

$$\frac{Dk}{Dt} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + \nu_t\left(\frac{\partial u}{\partial y}\right)^2 - \omega k \tag{3}$$

$$\frac{D\omega}{Dt} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial y}\right] + C_{\omega_1}\frac{\omega}{k}\left[\nu_t\left(\frac{\partial u}{\partial y}\right)^2\right] - C_{\omega_2}\omega^2 \tag{4}$$

The mean velocity, u is governed by

$$\frac{Du}{Dt} = \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial u}{\partial y}\right] \tag{5}$$

In the above equation, ν is the kinematic viscosity, σ_k, σ_ω, $C_{\omega 1}$, $C_{\omega 2}$ are model constants, and the subscript t designates a turbulent quantity. Using Equation (1) and setting k and ω as

$$k = U_s{}^2 h(\eta), \quad \omega = \frac{U_s}{l} g(\eta), \quad \nu_t = C_\mu \frac{k}{\omega} \tag{6}$$

the equations in the far wake reduce to a set of second order total differential equation in η. Two dimensionless parameters appear in these equations $\nu/U_s l$ and $U_s l/U_e x$. The first parameter was chosen over the range $10^{-4} - 10^{-6}$ The second parameter can be expressed as

$$\begin{aligned} \frac{U_e l}{U_s x} &= \frac{U_e}{U_s} \frac{l}{\theta} \frac{\theta}{x} \\ &= 8\sqrt{\pi ln(2)} \frac{\nu_t}{U_e \theta} \end{aligned} \tag{7}$$

where θ is the momentum thickness. As was indicated in Ref. [1], the general concensus is that $\nu_t/U_e\theta$ has a value of .032 when ν_t is assumed constant. In this work ν_t is variable. Because of this, the quantity $\nu_{to}/U_e\theta$, where ν_{to} is the value of ν_t at the line of symmetry, is assumed to be .032. The boundary conditions are

$$f'(\eta) = h'(\eta) = g'(\eta) = 0 \quad at \quad \eta = 0 \tag{8}$$

$$f(\eta) \rightarrow h(\eta) \rightarrow g(\eta) \rightarrow 0 \quad as \quad \eta \rightarrow \infty \tag{9}$$

Results and Discussion

The above system has four model constants. Because we are interested in model constants that are suited for flows over the body and its wake, the results of the log-law region are used to relate some of the model constants. The resulting equations can be written as

$$C_{\omega_2} - C_{\omega_1} = \frac{\kappa^2}{\sigma_\omega C_\mu^{\frac{1}{2}}} \tag{10}$$

where κ is the Karman constant and $C_\mu = .09$. Using Equation(10), the remaining constants were obtained by finding the "best" set of constants that reproduced the shear stress distribution, i.e.

$$G(\eta) = \frac{\tau}{\rho U_s{}^2} \tag{11}$$

and the velocity defect distribution, f(η). Because we are dealing with a two-point boundary value problem, the nature of the solutions obtained are shown in Figure (1). The selection of the model constants reduces to selecting the solution that has the shape labeled "correct solution" and gives a best fit of shear and defect velocity distributions. Since $\nu_{to}/U_e\theta$ is assumed to be .032, the values of g(0) and h(0) are related by the relation

$$\nu_{to} = C_\mu l U_s \frac{h(0)}{g(0)}$$

or,

$$\frac{\nu_{to}}{U_e\theta} = C_\mu \frac{lU_s}{U_e\theta}\frac{h(0)}{g(0)} \qquad (12)$$

The quantity $lU_s/U_e\theta$ is obtained from Ref. [1] as $[(4\pi)\,ln(2)]^2$

The approach outlined above is used to calculate the model constant in a $k-\omega$ model. Figure (2) shows the distribution of k, ($h(\eta)$) and ω, (g(η)). The shear stress distribution and velocity profiles are compared in figure (3) and (4) with the asymptotic solution, where ν_t is assumed constant throughout, and with the experiments of Ramaprian et al.[4] and Pot[5]. A summary of the constants obtained is given in Table 1. The table contains Wilcox's constants[6] along with constants developed by Anderson[7]. The results obtained by Anderson employed numerical optimization of computations starting on the flat plate and extending to $\frac{x}{\theta} = 2.5\text{x}10^3$ downstream of the plate trailing edge, and by comparison with asymptotic solution and available experiment.

Proposed Work

The next step in the research is to repeat the procedure for a $k-\epsilon$ model. The resulting sets of constants will then be used to calculate the flow field over the plate and its wake. Results will be compared with available experiment.

Acknowledgement

This work is supported in part by the following grants: NASA's Cooperative Agreement NCCI-22, and the Mars Mission Research Center funded by

NASA Grant NAGW-1331. The authors would like acknowledge many helpful discussions with Dr. E.C. Anderson, Senior Research Scientist NASA Langley Research Center.

References

[1] Patel, V.C., and Scheuerer, G., "Calculation of Two-Dimensional Near and Far Wakes" ,AIAA Journal, Vol. 20, No. 7, July, 1982, pp. 900-907.

[2] Patel, V.C., and Chen, H.C., "Turbulent Wake of a Flat Plate", AIAA Journal Vol. 25, No. 8, August, 1987, pp. 1078-1085.

[3] Tennekes, H. and Lumley, J.L., A First Course in Turbulence, The MIT Press., Cambridge, MA, 1989. Chapter 4.

[4] Ramaprian, B.R., Patel, V.C., and Sastry, M.S., "The symmetric Turbulent Wake of a Flat Plate", AIAA Journal, Vol. 20, No. 9, September, 1982, pp 1228-1235.

[5] Pot, P.J., "Measurement in a Two-Dimensional Wake and in a Two-Dimensional Wake Merging into a Boundary Layer", Data Report, NLR TR-79063 U, the Netherlands, 1979.

[6] Wilcox, D.C., "A complete Model of Turbulence Revisited", AIAA Paper 84-0176, January, 1984

[7] E.C. Anderson, Personal Communication.

Table 1. Comparison of Various Model Constants

Constant	Anderson	Wilcox	Current
C_{ω_1}	0.80	1.5667	0.49
C_{ω_2}	1.1733	1.8333	0.8633
C_{μ}	0.09	0.09	0.09
σ_k	1.17647	2.00	1.17647
σ_{ω}	1.42857	2.00	1.42857

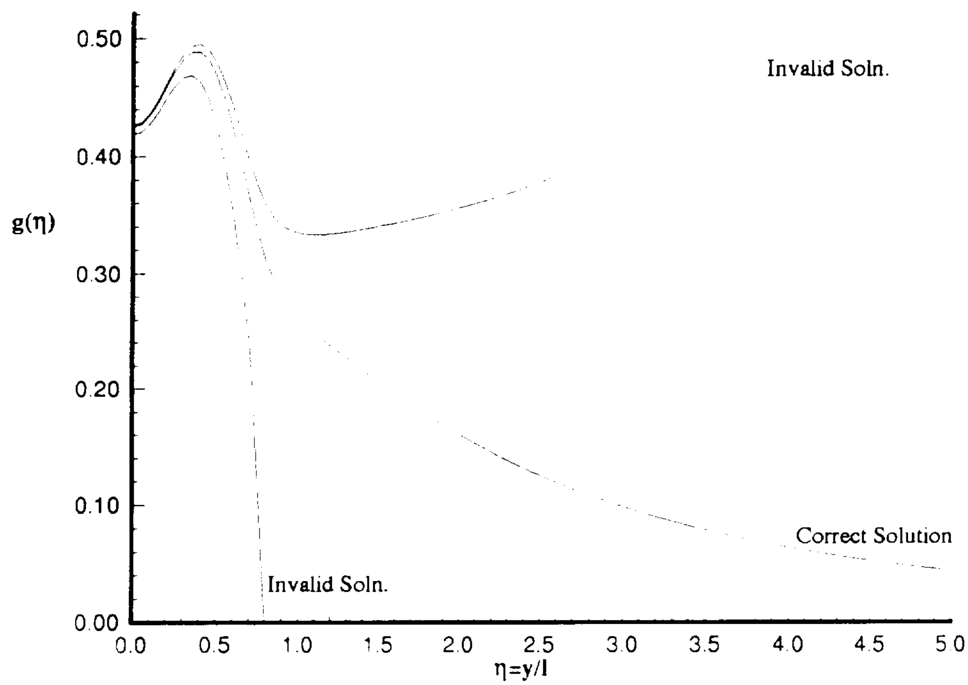

Figure 1. Method of Determining Paremeters

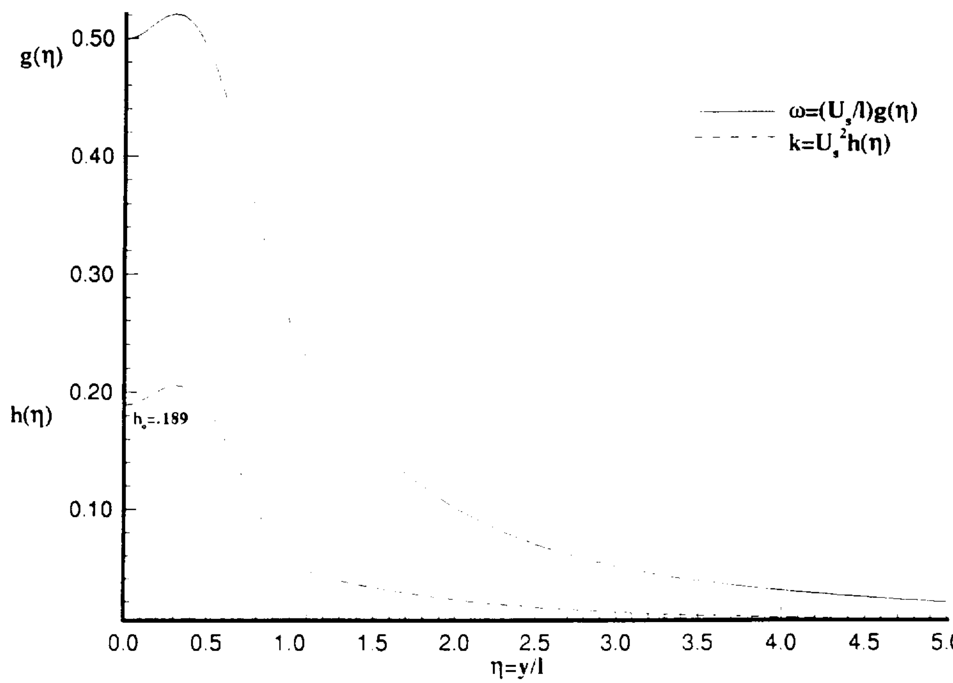

Figure 2. T.K.E. $h(\eta)$ and Frequency Scale $g(\eta)$ Profiles

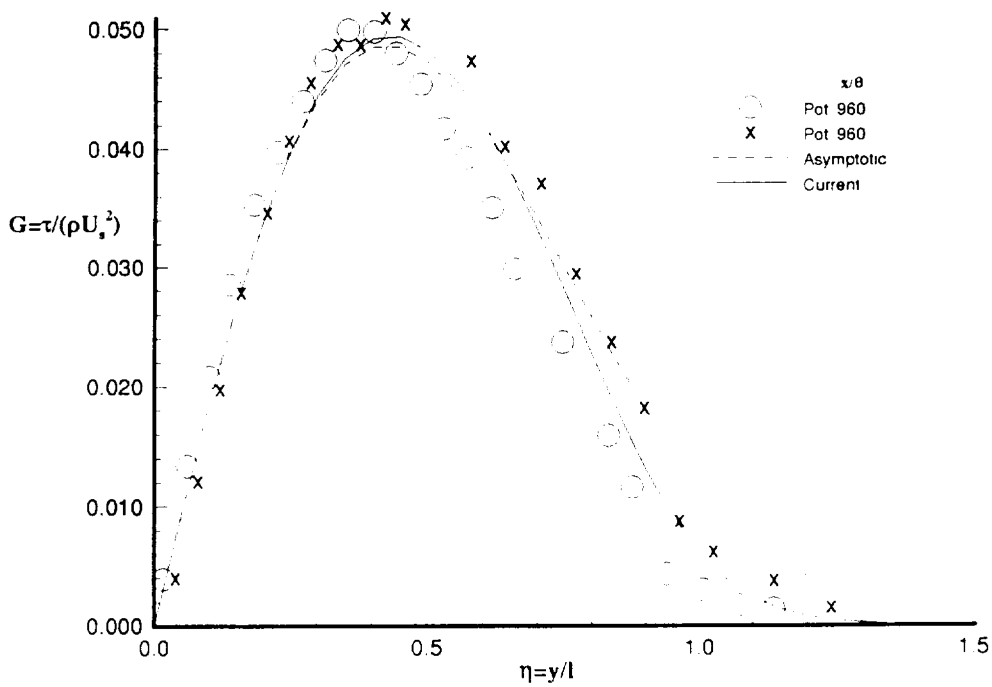

Figure 3. Shear Stress Profile, $G(\eta)$

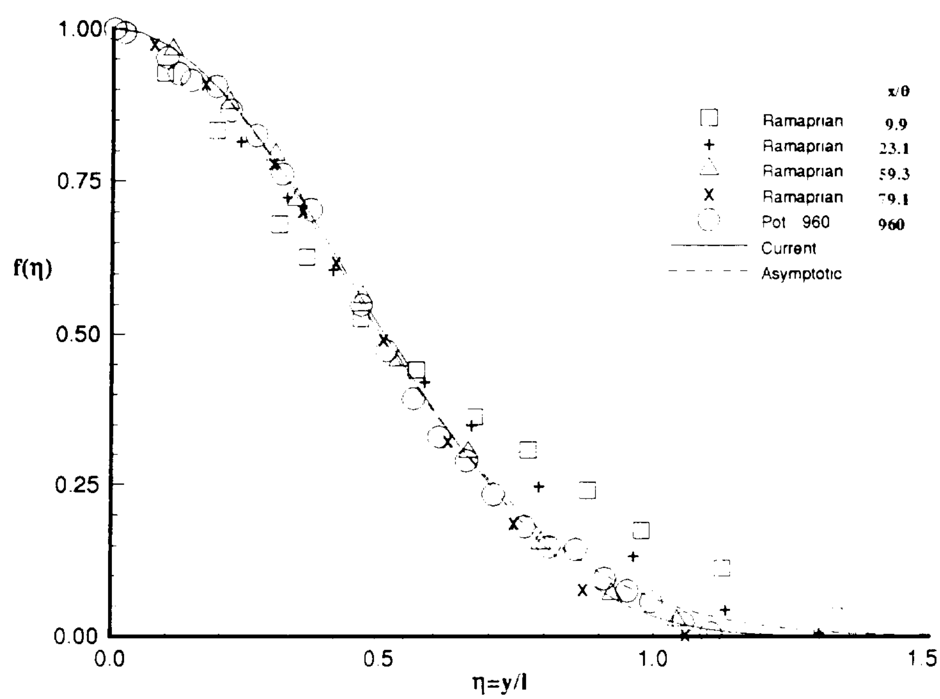

Figure 4. Velocity Defect Profile, $f(\eta)$